

SOME QUESTIONS OF EQUIVARIANT MOVABILITY

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ABSTRACT. In this article the some questions of equivariant movability connected with substitution of acting group G on closed subgroup H and with transitions to spaces of H -orbits and H -fixed points spaces are investigated. In the special case the characterization of equivariant-movable G -spaces is given.

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1. INTRODUCTION

The notion of movability was introduced by K. Borsuk [2]. Mardesic and Segal [17] have redefined movability in terms of ANR -systems. Movability can be defined in any *pro*-category. Moreover, recently the author of present article have defined the notion of movability in any category and have proved that the movability of topological space coincide with the movability of some suitable category [10].

The notion of equivariant movability or G -movability which was studied in the papers [9, 11, 12, 13, 6] one can define either in terms of ANR -systems [17] or by Borsuk's method [2]. We shall use both definitions of equivariant movability.

The aim of this paper is to study the connection between equivariant movability and movability. We prove that the G -movability implies the H -movability for any closed subgroup H of group G . In particular, the equivariant movability implies the ordinary movability of topological space. The converse is not true. The corresponding non simple example is constructed. Farther, we show that from the equivariant movability of G -space follows the movability of H -fixed points space and H -orbits space. In the case when the acting group G is a compact Lie group and the action is free, it is proved that the G -space X is equivariant movable if and only if the orbit space $X|_G$ is movable. We construct corresponding examples which show that in the last assertion the conditions of Lie group for group G and freeness of action are essential.

Let us denote the category of all metrizable spaces and continuous maps by M and the category of all p -paracompact spaces and continuous maps - by P . Corresponding equivariant categories we denote by M_G and P_G , respectively. Always in this article it is proposed that all topological spaces are p -paracompact and the group G is compact.

The reader is referred to the books by K. Borsuk [4] and by S. Mardesic and J. Segal [18] for general information about shape theory and to the book by G. Bredon [5] for introduction to compact transformation groups.

2. MOVABILITY AND EQUIVARIANT MOVABILITY

Let X be equivariant-movable G -space. The evident question emerges: does the movability of space X follows from their equivariant movability? The following, more general theorem gives positive answer (corollary 1) to the above question.

Theorem 1. *Let H be a closed subgroup of group G . Every G -movable G -space is H -movable.*

To prove this theorem the next one is important.

Theorem 2. *Let H be a closed subgroup of group G . Every $G-AR(P_G)$ ($G-ANR(P_G)$)-space is $H-AR(P_H)$ ($H-ANR(P_H)$)-space.*

Proof. According to the theorem of De Vries [7], it is sufficient to show that if X is p -paracompact H -space then the twisted product $G \times_H X$ is also p -paracompact. Indeed, since X and G are compacts, $G \times X$ is p -paracompact. Therefore the twisted product $G \times_H X$ is p -paracompact. \square

Proof of theorem 1. Let X be any equivariant-movable G -space. With respect to the theorem of Smirnov [21] there is a closed and equivariant embedding of G -space X to some $G-AR(P_G)$ -space Y . Let's consider all open G -invariant neighborhoods of type F_σ of G -space X in Y . By the theorem of R. Palais [20] this neighborhoods will form a cofinal family in the set of all open neighborhoods of X in Y , in particular, in the set of all open and H -invariant neighborhoods of H -space X in H -space Y , which is $H-AR(P_H)$ -space by theorem 1. Hence from G -movability of above mentioned family follows their H -movability. I.e. from G -movability of G -space X follows the H -movability of H -space X . \square

From the theorem 1 we obtain the following corollary if we consider the trivial subgroup $H = \{e\}$ of the group G .

Corollary 1. *Every equivariant movable G -space X is movable.*

The converse, in general, is not true, even if a cyclic group of order 2 Z_2 it takes as a group G (example 1).

3. MOVABILITY OF THE H -FIXED POINTS SPACE

Let X be G -space and H be closed subgroup of group G . The H -fixed points set of G -space X we denote by $X[H]$. Let us recall that $X[H] = \{x \in X; \quad hx = x, \text{ for } \forall h \in H\}$.

Theorem 3. *Let H be closed subgroup of group G . If G -space X be equivariant-movable, then the H -fixed points space $X[H]$ is movable.*

The proof requires the use of the following theorem.

Theorem 4. *Let H be closed subgroup of group G . Let X be $G - AR(P_G)(G - ANR(P_G))$ -space. Then H -fixed points space $X[H]$ is $AR(P)(ANR(P))$ -space.*

Proof. Let X be a $G - AR(P_G)(G - ANR(P_G))$ -space. By the theorem 1 it is sufficient to prove the theorem in the case $H = G$. I.e., we must prove that $X[G]$ is $AR(P)$ -space. By the theorem of Smirnov [21] we can consider X as a closed and G -equivariant subspace of a $G - AR(P_G)$ -space $C(G, V) \times \prod D_\lambda$ where V is a $AE(M)$ -space, $C(G, V)$ is a space of continuous maps from G to V with compact-open topology and with an action $(g'f)(g) = f(gg')$, $g, g' \in G, f \in C(G, V)$ of group G and D_λ is a closed ball of finite-dimensional Euclidean space E_λ with orthogonal action of group G .

At first, let us prove that the set $(C(G, V) \times \prod D_\lambda)[G]$ of all fixed points of G -space $C(G, V) \times \prod D_\lambda$ is $AR(P)$ -space. The spaces $C(G, V)$ and E_λ are normed-spaces. Since the actions of group G on $C(G, V)$ and E_λ are linear, the sets $C(G, V)[G]$ and $E_\lambda[G]$ will be closed and convex sets of local-convex spaces $C(G, V)$ and E_λ , respectively. Therefore by well known theorem of Kuratowski and Dugundji [3] $C(G, V)$ and E_λ are absolute retracts for metrizable spaces. By theorem of Lisica [15] they will be absolute retracts for p -paracompact spaces, too. The last conclusion is true for closed ball $D_\lambda \subset E_\lambda$ since the set $D_\lambda[G] = D_\lambda \cap E_\lambda[G]$ is closed and convex in E_λ .

Since the action of group G on product $C(G, V) \times \prod D_\lambda$ is coordinate-wise

$$(C(G, V) \times \prod D_\lambda)[G] = C(G, V)[G] \times \prod D_\lambda[G].$$

Hence, $(C(G, V) \times \prod D_\lambda)[G]$ is $AR(P)$ -space as a product of $AR(P)$ -spaces.

Now let's prove, that $X[G]$ is $AR(P)$ -space. Since X is $G - AR(P_G)$ -space it will be G -retract of product $C(G, V) \times \prod D_\lambda$. Therefore, $X[G]$

will be a retract of $AR(P)$ -space $(C(G, V) \times \prod D_\lambda)[G]$ and hence - $AR(P)$ -space.

The neighborhood case is proved similarly. \square

Proof of theorem 3. Let X be G -movable space. By the theorem 1 it is sufficient to prove the theorem in the case $H = G$. So, we must prove the movabilities of space $X[G]$ of all fixed points. The G -space X we consider as a closed and G -equivariant space of some $G-AR(P_G)$ -space Y [21]. The family of all open, G -invariant F_σ -type neighborhoods U_α of G -spaces X in Y , will be cofinal in the set of all open neighborhoods of X in Y [20]. It consists from $G-ANR(P_G)$ -spaces. The intersections $U_\alpha \cap Y[G] = U_\alpha[G]$ are $ANR(P)$ -spaces (theorem 4). They form a cofinal family of neighborhoods of space $X[G]$ in $Y[G]$. Indeed, for any neighborhood U of set $X[G]$ in $Y[G]$ there is a neighborhood V of the set $X[G]$ in Y such that $V \cap Y[G] = U$. Then the set $W = (Y \setminus Y[G]) \cup V$ will be a neighborhood of set X in Y , moreover $W \cap Y[G] = U$. There is an α such that $U_\alpha \subset W$ and therefor $U_\alpha[G] \subset U$. So the family of neighborhoods $U_\alpha[G]$ is cofinal.

Since X is G -movable for every U_α there is a neighborhood $U_{\alpha'} \subset U_\alpha$ that for any other neighborhood $U_{\alpha''} \subset U_{\alpha'}$ there exist G -equivariant homotopy $F : U_{\alpha'} \times I \rightarrow U_\alpha$ such that $F(y, 0) = y$ and $F(y, 1) \in U_{\alpha''}$ for any $y \in U_{\alpha'}$. It is not difficult to make sure that the induced by F homotopy $F[G] : U_{\alpha'}[G] \times I \rightarrow U_\alpha[G]$ satisfies the condition of movability of $X[G]$. \square

4. EXAMPLE FOR MOVABLE BUT NOT EQUIVARIANT MOVABLE SPACE

Example 1. Let us consider unit circle $S = \{z \in C; |z| = 1\}$. Let's denote $B = [S \times \{1\}] \cup [\{1\} \times S]$. B is a wedge of two copies of unit circle S with fixed point $\{1\}$. let's define continuous map $f : B \rightarrow B$ by formulas:

$$f(z, 1) = \begin{cases} (z^4, 1), & 0 \leq \arg(z) \leq \frac{\pi}{2} \\ (1, z^4), & \frac{\pi}{2} \leq \arg(z) \leq \pi \\ (z^{-4}, 1), & \pi \leq \arg(z) \leq \frac{3\pi}{2} \\ (1, z^{-4}), & \frac{3\pi}{2} \leq \arg(z) \leq 2\pi \end{cases}$$

$$f(1, t) = \begin{cases} (t^{-4}, 1), & 0 \leq \arg(t) \leq \frac{\pi}{2} \\ (1, t^{-4}), & \frac{\pi}{2} \leq \arg(t) \leq \pi \\ (t^4, 1), & \pi \leq \arg(t) \leq \frac{3\pi}{2} \\ (1, t^4), & \frac{3\pi}{2} \leq \arg(t) \leq 2\pi \end{cases}$$

for every z and t from S . Let us consider ANR -sequences

$$B \xleftarrow{f} B \xleftarrow{f} B \xleftarrow{f} \dots$$

and

$$\Sigma B \xleftarrow{\Sigma f} \Sigma B \xleftarrow{\Sigma f} \Sigma B \xleftarrow{\Sigma f} \dots$$

where Σ is a operation of suspension. Let's denote

$$P = \varprojlim \{B, f\}.$$

Then

$$\Sigma P = \varprojlim \{\Sigma B, \Sigma f\}.$$

Let's define an action of group $Z_2 = \{e, g\}$ on ΣB by formulas

$$e[x, t] = [x, t]; \quad g[x, t] = [x, -t].$$

for every $[x, t] \in \Sigma B$, $-1 \leq t \leq 1$. It induces an action on ΣP .

Proposition 1. *The space ΣP has trivial shape but not Z_2 -movable.*

Proof. The triviality of shape of space ΣP is proved by method of Mardesic [16]. Let's prove that the space ΣP is not Z_2 -movable. Consider the set $\Sigma P[Z_2]$ of all fixed-points of Z_2 -space ΣP . It is obvious that $\Sigma P[Z_2] = P$. Hence, by theorem 3 it is sufficient to prove the following proposition. \square

Proposition 2. *The space P is not movable.*

Proof. Since the movability of inverse system is unchanged during the functorial transitions, it is sufficient to prove the non-movabilities of inverse sequence of groups

$$(1) \quad \pi_1(B) \xleftarrow{f_*} \pi_1(B) \xleftarrow{f_*} \pi_1(B) \xleftarrow{f_*} \dots$$

where $\pi_1(B)$ is fundamental group of space B and f_* is a homomorphism induced by mapping $f : B \rightarrow B$.

It is known that for sequences of groups the movability is equivalent to the following condition of Mittag-Leffler [20]:

The inverse system $\{G_\alpha, p_{\alpha\alpha'}, A\}$ of pro- $GROU$ P category is movable if and only if the following condition, which called the Mittag-Leffler condition, is hold:

(ML) For every $\alpha \in A$ there exist $\alpha' \in A$, $\alpha' \geq \alpha$ such that $p_{\alpha\alpha'}(G_{\alpha'}) = p_{\alpha\alpha''}(G_{\alpha''})$ for any $\alpha'' \in A$, $\alpha'' \geq \alpha$ [20].

Thus we must prove that the sequence (1) does not satisfy the condition (ML). Let's observe that $\pi_1(B)$ is a free group with two generators a and b , and f_* is homomorphism defined by formulas

$$f_*(a) = aba^{-1}b^{-1}, \quad f_*(b) = a^{-1}b^{-1}ab.$$

It is easy to verify that f_* is monomorphism but not epimorphism. Hence for any natural m and n $Imf_*^m \subsetneq Imf_*^n$ if only $m > n$. It means that the inverse sequence (1) does not satisfy condition (ML). \square

5. MOVABILITY OF ORBIT SPACE

Let X be any G -space and H be a closed and normal subgroup of group G . The set $Hx = \{hx; h \in H\}$ is called an H -orbit of point $x \in X$. Let's denote by $X|_H$ the H -orbits space of G -space X . There is an action of group G on the space $X|_H$ defined by formula $gHx = Hgx$, $g \in G, x \in X$. So, $X|_H$ is a G -space.

Theorem 5. *Let X be metrizable G -space. If X is G -movable then for any closed and normal subgroup H of group G the H -orbit space $X|_H$ is G -movable too.*

Proof. Without losing generality one may suppose that X is closed and G -invariant subset of some $G-AR(M_G)$ -space Y [1]. It is obvious that $X|_H$ is closed and G -invariant subset in $Y|_H$ which is $G-AR(M_G)$ -space.

Let $\{X_\alpha, \alpha \in A\}$ be family of all G -invariant neighborhoods of X in Y . Let us consider the family $\{X_\alpha|_H, \alpha \in A\}$, where each $X_\alpha|_H \in G-ANR(M_G)$ and is G -invariant neighborhood of $X|_H$ in $Y|_H$. Let's prove that the family $\{X_\alpha|_H, \alpha \in A\}$ is cofinal in the family of all neighborhoods of $X|_H$ in $Y|_H$. Let U be arbitrary neighborhood of $X|_H$ in $Y|_H$. By one theorem of Palais [20] there exist G -invariant neighborhood $V \supset X|_H$ laying in U . Let's denote $\tilde{V} = pr^{-1}(V)$ where $pr : Y \rightarrow Y|_H$ is H -orbit projection. It is evidently that \tilde{V} is G -invariant neighborhood of space X in Y and $V = \tilde{V}|_H$. So in any neighborhood of space $X|_H$ in $Y|_H$ there is the neighborhood of type $X_\alpha|_H$ where X_α - G -invariant neighborhood of X in Y .

Now let's prove the G -movability of space $X|_H$. Let X be G -movable. It means that the inverse system $\{X_\alpha, i_{\alpha\alpha'}, A\}$ is G -movable. We must prove that the induced inverse system $\{X_\alpha|_H, i_{\alpha\alpha'}|_H, A\}$ is G -movable. Let $\alpha \in A$ be any index. By G -movability of inverse system $\{X_\alpha, i_{\alpha\alpha'}, A\}$ there is $\alpha' \in A$, $\alpha' > \alpha$ such that for any other index $\alpha'' \in A$, $\alpha'' > \alpha$ there exist G -mapping $r^{\alpha'\alpha''} : X_{\alpha'} \rightarrow X_{\alpha''}$ which

makes the following diagram G -homotopic commutative

$$\begin{array}{ccc}
 & X_{\alpha'} & \\
 i_{\alpha\alpha'} \swarrow & & \searrow r^{\alpha'\alpha''} \\
 X_{\alpha} & & X_{\alpha''} \\
 & i_{\alpha\alpha''} \swarrow &
 \end{array}$$

Diagram 1.

It turn out that for given $\alpha \in A$ the obtained index $\alpha' \in A$, $\alpha' > \alpha$ satisfies a condition of G -movability of inverse system $\{X_{\alpha}|_H, i_{\alpha\alpha'}|_H, A\}$, too. It is obvious since from G -homotopic commutativity of diagram 1 follows the G -homotopic commutativity of the next diagram

$$\begin{array}{ccc}
 & X_{\alpha'}|_H & \\
 i_{\alpha\alpha'}|_H \swarrow & & \searrow r^{\alpha'\alpha''}|_H \\
 X_{\alpha}|_H & & X_{\alpha''}|_H \\
 & i_{\alpha\alpha''}|_H \swarrow &
 \end{array}$$

Diagram 2.

where $r^{\alpha'\alpha''}|_H : X_{\alpha'}|_H \rightarrow X_{\alpha''}|_H$ is induced by mapping $r^{\alpha'\alpha''}$.

So, the G -movability of space $X|_H$ is proved. \square

Corollary 2. *Let X be metrizable G -space. If X G -movable then the orbit space $X|_G$ is movable.*

Proof. In the case $H = G$ from the last theorem we obtain that the orbit space $X|_G$ with the trivial action of group G is G -movable. Therefore it will be movable by corollary 1. \square

The corollary 2 in general is not convertible:

Example 2. Let Σ be a solenoid. It is known [4] that Σ is non-movable, compact, metrizable Abelian group. By corollary 1 the solenoid Σ with the natural group action not Σ -movable although the orbit space $\Sigma|_{\Sigma}$ is movable as a one-point set.

The converse of corollary 2 is true if the group G is Lie group and action is free (theorem 7).

6. EQUIVARIANT MOVABILITY OF FREE G -SPACE

Let's recall some notions of topological transformation groups. Let X be G -space. The set $G_x = \{g \in G; \quad g(x) = x\}$ is a closed subgroup of group G and is called a stationary subgroup (or a stabilizer) of a point x . The action of group G on X (or the G -space X) is called free if a stationary subgroup G_x is trivial for any $x \in X$. They say that the orbits Gx and Gy of points x and y , respectively, have the same type if the stationary subgroups G_x and G_y are conjugate.

Theorem 6. *Let G be a compact Lie group and Y be metrizable $G - AR(M_G)$ -space. Suppose that the close and invariant subset X of Y has invariant neighborhood which orbits have the same type. If the orbit space $X|_G$ is movable then X is equivariant-movable.*

Proof. The orbit space $X|_G$ is closed in $Y|_G$ which is $G - AR(M)$ -space. Let U be arbitrary invariant neighborhood of X in Y . By assumption of theorem it follows that there exist cofinal family of neighborhoods of X in Y which orbits have the same type. Therefore one may suppose that all orbits of neighborhood U have the same type. The orbit set $U|_G$ will be a neighborhood of $X|_G$ in $Y|_G$. From movabilities of $X|_G$ follows that for neighborhood $U|_G$ there is a neighborhood \tilde{V} of space $X|_G$ in $Y|_G$ which lies in neighborhood $U|_G$ and where it contracts to any predesigned neighborhood of space $X|_G$.

Let's denote $V = pr^{-1}(\tilde{V})$ where $pr : Y \rightarrow Y|_G$ is orbit projection. It is evident that V is invariant neighborhood of space X lying in U . Let's prove that V in U contracts to any predesigned invariant neighborhood of X . Let W be any invariant neighborhood of X in Y . We must prove the existing of equivariant homotopy $F : V \times I \rightarrow U$ which satisfies a condition

$$F(x, 0) = x, \quad F(x, 1) \in W$$

for any $x \in V$. Since $W|_G$ is neighborhood of space $X|_G$ in $Y|_G$ there is homotopy $\tilde{F} : V|_G \times I \rightarrow U|_G$ such that

$$(2) \quad F(\tilde{x}, 0) = \tilde{x}, \quad \tilde{F}(\tilde{x}, 1) \in W|_G$$

for any $\tilde{x} \in V|_G$. The homotopy $\tilde{F} : V|_G \times I \rightarrow U|_G$ keeps the G -orbit structure, because $V \subset U$ and all orbits of U have the same types. Let's consider

$$\begin{array}{ccc}
V & \xrightarrow{i} & U \\
pr \downarrow & & \downarrow pr \\
V|_G & \xrightarrow{i'} & U|_G
\end{array}$$

Diagram 3.

$\tilde{F} : V|_G \times I \rightarrow U|_G$ is homotopy of including $i' : V|_G \rightarrow U|_G$, keeping the orbit structure. By covering homotopy theorem of Palais [20] there is an equivariant homotopy $F : V \times I \rightarrow U$ which covers homotopy \tilde{F} and satisfies $F(x, 0) = i(x) = x$. That is one have a commutative diagram

$$\begin{array}{ccc}
V \times I & \xrightarrow{F} & U \\
pr \downarrow & & \downarrow pr \\
V|_G \times I & \xrightarrow{\tilde{F}} & U|_G
\end{array}$$

Diagram 4.

The equivariant homotopy $F : V \times I \rightarrow U$ is unknown. For this it remains to verify that $F(x, 1) \in W$. But it immediately follows from (2) and commutativity of diagram 4. \square

Theorem 7. *Let G be a compact Lie group. Metrizable free G -space X is equivariant-movable if and only if the orbit space $X|_G$ is movable.*

Proof. The necessity in more general case was proved in corollary 2. Let's prove the sufficiency. Let the orbit space $X|_G$ be movable. The G -space X one can consider as a closed and invariant subset of some $G - AR(M_G)$ -space Y . Let $P \subset X$ be any orbit. From the existence of slice follows that around P there is such invariant neighborhood $U(P)$ in Y that $type Q \geq type P$ for any orbit Q from $U(P)$ [5]. Since the action of group G on X is free $type Q = type P = type G$ for any orbit Q lying in $U(P)$. Let's denote $V = \cup\{U(P); P \in X|_G\}$. It is evident that V is invariant neighborhood of space X in Y and that all it's orbits have the same type. Hence, by theorem 6 X is equivariant-movable. \square

The example 2 shows that the condition of Lie group for group G is essential in the above theorem. The below example 3 shows that the condition of freeness of action of group G is essential in above theorem, too.

7. EXAMPLE FOR NON FREE Z_2 -NONMOVABLE SPACE WITH
MOVABLE ORBIT SPACE

Example 3. Let's consider the space $P = \varprojlim \{B, f\}$ constructed in example 1. Let's define an action of group $Z_2 = \{e, g\}$ on space B by formulas

$$(3) \quad \begin{aligned} e(z, 1) &= (z, 1) \\ e(1, t) &= (1, t) \\ g(z, 1) &= (1, z^{-1}) \\ g(1, t) &= (t^{-1}, 1) \end{aligned}$$

for any z and t from S . B is Z_2 - $ANR(M_{Z_2})$ space [19] with fixed-point $b_0 = (1, 1)$.

Proposition 3. *The mapping $f : B \rightarrow B$ defined by formulas (3) is equivariant.*

Proof. It is necessary to prove the following two equalities:

$$(4) \quad \begin{aligned} f(g(z, 1)) &= g(f(z, 1)) \\ f(g(1, t)) &= g(f(1, t)) \end{aligned}$$

for any z and t from S . Let's prove the first. Consider some cases:

$$\text{Case 1. } 0 \leq \arg z \leq \frac{\pi}{2} \Leftrightarrow \frac{3\pi}{2} \leq \arg z^{-1} \leq 2\pi.$$

$$\text{Then } f(g(z, 1)) = f(1, z^{-1}) = (1, z^{-4}) = g(z^4, 1) = gf(z, 1).$$

$$\text{Case 2. } \frac{\pi}{2} \leq \arg z \leq \pi \Leftrightarrow \pi \leq \arg z^{-1} \leq \frac{3\pi}{2}.$$

$$\text{Then } f(g(z, 1)) = f(1, z^{-1}) = (z^{-4}, 1) = g(1, z^4) = gf(z, 1).$$

$$\text{Case 3. } \pi \leq \arg z \leq \frac{3\pi}{2} \Leftrightarrow \frac{\pi}{2} \leq \arg z^{-1} \leq \pi.$$

$$\text{Then } f(g(z, 1)) = f(1, z^{-1}) = (1, z^4) = g(z^{-4}, 1) = gf(z, 1).$$

$$\text{Case 4. } \frac{3\pi}{2} \leq \arg z \leq 2\pi \Leftrightarrow 0 \leq \arg z^{-1} \leq \frac{\pi}{2}.$$

$$\text{Then } f(g(z, 1)) = f(1, z^{-1}) = (z^4, 1) = g(1, z^{-4}) = gf(z, 1).$$

In this manner the second equality of (4) is proved. \square

Proposition 4. *P is connected, compact, metrizable and equivariant non-movable Z_2 -space which free in all points except fixed-point (b_0, b_0, \dots) and with $sh(P|_{Z_2})=0$.*

Proof. P is Z_2 -space as a inverse limit of Z_2 - $ANR(M_{Z_2})$ -spaces B and equivariant mappings f . The uniqueness of fixed-point is evident. The connectedness, compactness and metrizability follows from the properties of inverse systems [8]. Z_2 -nonmovability follows from proposition 2 and corollary 1.

Let's prove that $sh(P|_{Z_2}) = 0$ from which, in particular, follows the movability of orbit space $P|_{Z_2}$.

Let $X = \varprojlim \{B|_{Z_2}, f|_{Z_2}\}$. X is equimorphic to the orbit space $P|_{Z_2}$. Indeed, let's define a mapping $h : X \rightarrow P|_{Z_2}$ in the following way:

$$h([x_1], [x_2], \dots) = [(x_1, x_2, \dots)]$$

where $([x_1], [x_2], \dots) \in X$, and x_1, x_2, \dots are selected from the classes $[x_1], [x_2], \dots$ in such way that $(x_1, x_2, \dots) \in P$ or what is the same $f(x_{n+1}) = x_n$ for any $n = 1, 2, \dots$. Let's prove that the mapping h is defined correctly. Let $\tilde{x}_1, \tilde{x}_2, \dots$ be some other representatives of classes $[x_1], [x_2], \dots$, respectively, with the conditions $f(\tilde{x}_{n+1}) = \tilde{x}_n$ for any $n \in N$. Since each class $[x_n]$ has two representatives: x_n and gx_n where $g \in Z_2 = \{e, g\}$ hence either $\tilde{x}_n = gx_n$ or $\tilde{x}_n = x_n$. But it is obvious that if for some $n_0 \in N$ $\tilde{x}_{n_0} = gx_{n_0}$ then for any $n \in N$, $\tilde{x}_n = gx_n$ by equivariantness of f . Thus in case of another choice of representatives of classes $[x_1], [x_2], \dots$ we have

$$\begin{aligned} h([x_1], [x_2], \dots) &= [(\tilde{x}_1, \tilde{x}_2, \dots)] = [(gx_1, gx_2, \dots)] = \\ &= [g(x_1, x_2, \dots)] = [(x_1, x_2, \dots)]. \end{aligned}$$

h is bijection and continues and so - homeomorphism [8].

Thus

$$P|_{Z_2} = \varprojlim \{B|_{Z_2}, f|_{Z_2}\},$$

where, as it is not difficult to see, $B|_{Z_2} \cong S$ and the mapping $\bar{f} = f|_{Z_2} : S \rightarrow S$ defined by formulas:

$$(5) \quad \bar{f}(z) = \begin{cases} z^4, & 0 \leq \arg(z) \leq \frac{\pi}{2} \\ z^{-4}, & \frac{\pi}{2} \leq \arg(z) \leq \frac{3\pi}{2} \\ z^4, & \frac{3\pi}{2} \leq \arg(z) \leq 2\pi \end{cases}$$

for any $z \in S$. Thus, we have obtained that the orbit space $P|_{Z_2}$ is a limit of inverse sequence

$$S \xleftarrow{\bar{f}} S \xleftarrow{\bar{f}} S \xleftarrow{\bar{f}} \dots$$

By formula (5) the mapping \bar{f} induces a homomorphism $\bar{f}_* : \pi_1(S) \rightarrow \pi_1(S)$, which acts in the next way:

$$\bar{f}_*(a) = aa^{-1}a^{-1}a,$$

where $a \in \pi_1(S) \cong Z$ is generator of group Z . From the above formula it follows that \bar{f}_* is a null-homomorphism and hence $\deg \bar{f} = 0$. For any $k = 1, 2, \dots$ \bar{f}_*^k is also null-homomorphism and hence $\deg \bar{f}^k = 0$. Therefore by Hopf classic theorem [14] all $\bar{f}^k : S \rightarrow S$ are null-homotopic and so $sh(P|_{Z_2}) = 0$. \square

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